

The CST Bounce Universe model – a parametric study

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Abstract

A bounce universe with a scale invariant as well as stable spectrum of primordial density perturbations was constructed from a string theoretical elements, using a tachyon effective potential arisen from the D-brane and anti-D-brane system. The tachyon is stabilized at the top of its potential via a coupling to the Higgs field within. This is called the Coupled Scalar and Tachyon Bounce Universe, or CSTB cosmos for short. This bounce universe has been shown to be free of ghosts and does not violate the null energy condition. It can also solve the Big Bang cosmic singularity problem. In this article we follow up with a thorough study of the parameter space of the CSTB model. In particular we are interested in the parameter space that can produce a single bounce to arrive at a radiation dominated universe. We further establish that CSTB universe is a viable alternative to inflation, as it can naturally produce enough e-foldings—in the pre-bounce contractional phase as well in the post bounce expanding phase—to solve the flatness, horizon and homogeneity problems of the Big Bang model, leading to an observed universe of the current size.

KEYWORDS: bounce universe, big bang singularity, tachyon inflation, flatness and horizon problems

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Introduction

The three most fundamental problems in the Big Bang theory of cosmology are the flatness problem, the horizon problem and the magnetic monopole problem. In 1980s inflation was proposed [1] to solve them. The predictions of inflation paradigm are consistent with observational facts, such as the highly isotropic Cosmic Microwave Background Radiation (CMBR), and the scale invariance of primordial density perturbations attributed to the quantum fluctuations of the inflaton field at the end of inflation [13]. Inflation, however, is not free of problems. For instance, the initial singularity does not disappear but is delayed [3].

Building on the success of inflation models, cosmologists have been searching for alternative scenarios that also address the problem of the cosmic singularity. In 1993 a GR-compatible solution utilizing a nonsingular bounce was proposed shortly after the first observation of current expansion [14]. At the same time, string theorists attempted to implement a nonsingular bounce in dilaton gravity, leading to the pre-big bang (PBB) scenario within string theory framework [10]. And many string inflation models as reviewed by Linde [4], and by Tye [5].

Bounce universe has a long history, dated back to Einstein, and many models [15]. A period of contraction is postulated to happen before the period of expansion resulting our current observable universe. For instance, a cyclic model in a conventional 4-dimensional quantum field theory generates a bounce which solved the singularity problem [16]. A “matter bounce” driven by ghost condensation was proposed to avoid ghost in the perturbation spectrum of states [17]; a bounce model in $N=1$ super gravity was proposed to get rid of ghost excitations [18] and to prove that nonsingular bounces were viable in supergravity. A “G-Bounce” model, which generates a non-singularity inflation cosmos with a Galileon field [19], produces a bounce in the Lee-Wick radiation phase with an apparent scale invariance in the perturbation spectrum [20]. A bounce solution was also found in the universe dominated by the Quintom matter [21]. There is also the famous Ekpyrotic universe [22] inspired by D-branes dynamics. Bounce conditions were also found possible in $f(R)$ cosmology in [56].

1 The Single Bounce Criteria

In a string model with the coupled scalar and tachyon fields—the CSTB model—a bouncing universe happens naturally with a generic set of parameters. In the effective theory, a D- $\bar{D}3$ -brane pair is described by the open

string tachyon field action [23],

$$\mathcal{L}_T = V_T \sqrt{1 + M_s^{-4} \partial_\mu T \partial^\mu T}, \quad V_T = \frac{V_0}{\cosh(\frac{T}{\sqrt{2}M_s})}$$

where T , M_s , V_0 are the tachyon field, the string mass, the tension of D- \bar{D} 3-brane pair respectively. The tachyon potential V_T has a maximum at $T = 0$ and two minima at $T \rightarrow \pm\infty$. We take the background metric as $(-, +, +, +)$, and we also assume that the tachyon field is spatially homogeneous.

In D-brane inflation[24] the attractive potential of the D- \bar{D} pair takes the following form:

$$V_\phi = \frac{1}{2} m_\phi^2 \phi^2 + V_0 - \frac{V_0^2}{4\pi^2 v \phi^4}, \quad \phi \equiv \sqrt{V_0} y,$$

where y being the distance between the D- \bar{D} 3-branes. The CSTB model[6] introduces a scalar-tachyon coupling term $\lambda \phi^2 T^2$, and the tachyon condensation comes about naturally after the locked inflation driven by the tension of D- \bar{D} 3-branes. With the coupling term playing a crucial role in the cosmic evolution the Lagrangian takes the form:

$$\mathcal{L} = \mathcal{L}_T - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 - \lambda \phi^2 T^2,$$

in addition to the Hilbert-Einstein term, in a FRW background with $k = 1$,

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

in order not to violate the null energy condition[27].

1.1 Cosmic evolution of the CSTB cosmos

In the CSTB bounce universe model, the universe undergoes a contraction phase, deflation, the “bounce,” and then the locked inflation² at the peak of the tachyon potential, and the subsequent tachyon-matter-dominated rolling expansion[25][26] as depicted in Fig.1. The background cosmology evolves according to the Friedmann equation,

$$H^2 = -\frac{1}{a^2} + \frac{8\pi}{3M_p^2} \left[\frac{V(T)}{\sqrt{1 - M_s^{-4} \dot{T}^2}} + \left(\frac{1}{2} m_\phi^2 + \lambda T^2 \right) \phi^2 + \frac{1}{2} \dot{\phi}^2 \right];$$

²Coupling was introduced in[6] to address the problems with tachyon inflations[28].

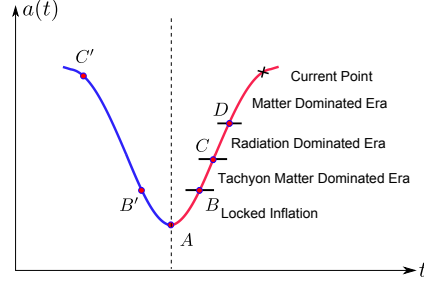


Figure 1: The evolution of cosmos in the CSTB model. Our model starts from C' as it undergoes a contraction phase, C' to B' , and then a deflation era from B' to A , before entering a locked inflation era from A to B . At point A , the bounce point, at which the exponential expansion of the universe starts, $\dot{a} = 0$ and $\ddot{a} > 0$. The tachyon condensation takes place at B , where all the energy of tachyon field transfers to tachyon matter. The phase transition at B is followed by tachyon matter dominated period, $B \rightarrow C$. At point C' , reheating may happen as the universe undergoes another phase transition, during which the energy of tachyon matter transfers to radiation.

whereas the equations of motion for T and ϕ are governed by

$$\ddot{T}^2 + (1 - \dot{T}^2) \left[3H\dot{T} + \frac{V'(T) + 2\lambda\phi^2 T \sqrt{1 - \dot{T}^2}}{V(T)} \right] = 0,$$

$$\ddot{\phi} + 3H\dot{\phi} + (m_\phi^2 + 2\lambda T^2)\phi = 0;$$

where the string mass, M_s , has been suppressed.

1.2 How to make a successful bounce: initial conditions

In this section we study the constraints on parameters required by enough e-foldings. In the locked inflation era, the universe is dominated by vacuum energy. The Hubble parameter is nearly constant as long as the tachyon is locked at its peak by the scalar field ϕ . The dynamic equations of the Hubble parameter, ϕ and T can be simplified as follows,

$$H^2 - \frac{8\pi V_0}{3M_p^2} = 0 \quad (1)$$

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi = 0 \quad (2)$$

$$\frac{1}{M_s^4}\ddot{T} + \left(-\frac{1}{2M_s^2} + \frac{2\lambda\phi^2}{V_0} \right) T = 0. \quad (3)$$

The scalar field, ϕ , oscillates while being redshifted

$$\phi \propto a^{-\frac{3}{2}} e^{i\sqrt{m_\phi^2 - \frac{9}{4}H^2}t}. \quad (4)$$

We can calculate the number of e-folds N_L in this era determined by the critical value of $\langle\phi_c^2\rangle = V_0/4\lambda M_s^2$:

$$N_L = \frac{1}{6} \ln \frac{32\lambda^3 M_s^4 \langle\phi_0^2\rangle^2 \langle T_0^2\rangle}{m^2 V_0^2}. \quad (5)$$

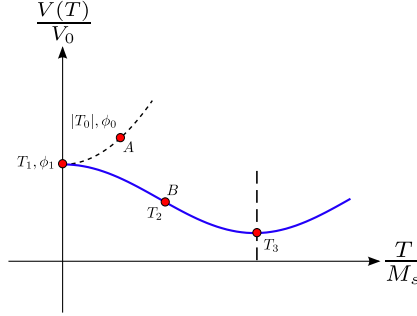


Figure 2: A generic cross section ($\phi \neq 0$) of the tachyon-scalar field space: During the contraction period the tachyon field, T , blue-shifts from T_3 to T_2 as the universe contracts from Point C' to Point B' in Fig.1. When T field goes to T_1 , the effective mass of T becomes positive (dash line from T_1 to T_0), the universe is dominated by vacuum energy and undergoes a deflation until the Hubble parameter becomes zero. When $H = 0$, the tachyon field is locked at T_0 , and the universe starts to inflate.

Let us now turn to the observational constraints can be imposed on our bounce model.

- *A period of vacuum energy domination:* The first constraint comes from vacuum energy domination. In the locked inflation era, the tension of D- \bar{D} -brane is so large that it dominates the energy density in this era.

$$V_0 > \frac{1}{2} m^2 \langle\phi_0^2\rangle + \lambda \langle T_0^2\rangle \langle\phi_0^2\rangle \quad (6)$$

- *The bounce point:* For an extended period of locked inflation at $T \sim 0$, the initial value $\langle\phi_0^2\rangle$ should be larger than $\langle\phi_1^2\rangle$. And the locked inflation ends when

$$\langle\phi_0^2\rangle < \frac{V_0}{4\lambda M_s^2} \quad (7)$$

- *The effective mass of ϕ :* The effective mass of ϕ is given by $m_{eff} = m_\phi^2 + 2\lambda T^2$. During locked inflation, T red-shifts and its *vev* $\langle T^2 \rangle$ diminishes dramatically. We assume that at the beginning of the locked inflation era T_0 is very large and the T field is smaller than the bare mass of scalar field, m_ϕ at the end; therefore

$$2\lambda\langle T_0^2 \rangle > m_\phi^2 > 2\lambda\langle T_1^2 \rangle \quad (8)$$

and $\langle T_1^2 \rangle$ can be expressed as $\sqrt{\frac{m_\phi^2 V_0 \langle T_0^2 \rangle}{8\lambda^2 M_s^2 \langle \phi_0^2 \rangle}}$.

- *Keeping ϕ oscillating:* To keep ϕ oscillating in the locked inflation era, we demand the frequency of ϕ in 4 be real, which in turn implies $m_\phi^2 \phi > \frac{9}{4}H^2$.

$$2\lambda\langle T_0^2 \rangle > \frac{6\pi}{M_p^2}(V_0 + 2\lambda\langle T_0^2 \rangle \langle \phi_0^2 \rangle). \quad (9)$$

With these constraints we deduce the two critical relations amongst the key parameters m_ϕ , M_s and λ :

$$0 < \frac{m}{\lambda M_p} < \frac{4(\sqrt{2}-1)}{\sqrt{3\pi}} \sim 0.54, \quad (10)$$

$$\lambda > \frac{1}{8}. \quad (11)$$

We thus see that these values cover a wide area of the parameter space, and it is easy to pick a generic set of initial conditions for a successful bounce model resulting in a realistic universe provided that $N_L \sim 6$ e-foldings are generated in the locked inflation era. This is to be contrasted with the fine tuning one needs for inflation models.

2 The contraction era in CSTB Universe

We shall show that CSTB universe is free of the Horizon problem by calculating the e-foldings in the contraction phase of the CSTB universe.

2.1 CSTB model solves the horizon problem

Bouncing universe models solve the horizon problem because there exists a phase of contraction—prior to the bounce—long enough to put the entire universe back into thermal contact and re-establish causality. In CSTB model, particle horizon is therefore nonzero. Considering that particle horizons in

expansion and contraction phases are symmetric, we only calculate the particle horizon in expansion phase.

In the era of locked inflation, the universe is vacuum energy dominated, thus the Friedman equation becomes equation 1. The universe undergoes exponential expansion as:

$$a = a_0 e^{\sqrt{\frac{8\pi V_0}{3M_p^2}} t}. \quad (12)$$

The particle horizon in locked inflation is given by

$$d_{p_1} = a_1 \int_{a_0}^{a_1} \frac{da}{Ha^2} = \frac{a_1}{H_0} \left(\frac{1}{a_0} - \frac{1}{a_{max}} \right). \quad (13)$$

Shortly after the bounce the Hubble parameter changes from 0 to $\sqrt{\frac{8\pi V_0}{3M_p^2}}$, and $H_0 a_0 = 1$, which in turn implies

$$d_{p_1} = a_1 - a_0.$$

In the tachyon matter dominated era, the growth of the particle horizon is

$$d_{p_2} = a_{max} \int_{a_1}^{a_{max}} \frac{da}{Ha^2} = \frac{2a_0 a_{max}}{a_1^{\frac{3}{2}}} (\sqrt{a_{max}} - \sqrt{a_1}).$$

All in all the total particle horizon at the end of expansion would be

$$d_{p_{tot}} = d_{p_1} + d_{p_2} = a_1 - a_0 + \frac{2a_0 a_{max}}{a_1^{\frac{3}{2}}} (\sqrt{a_{max}} - \sqrt{a_1});$$

therefore $\frac{d_{p_{tot}}}{a_0} \gg 1$. Since the maximum size of universe is larger than current size a_0 at the bounce point the particle horizon is much larger than the scale factor. So bouncing universe models guarantee that the particle horizon be larger than the current homogeneous regions.

2.2 E-foldings of the final universe

In Big Bang cosmos the flatness problem is a cosmological fine-tuning problem. CMB data[2] determines the current universe to be flat up to 1% percent level, $\Omega = 1.00 \pm 0.01$, or

$$\frac{1}{a_{now}^2 H_{now}^2} < 0.01. \quad (14)$$

As shown in Fig. 1 we can read off the required e-foldings from the end of locked inflation to the present:

$$\begin{aligned} N_r = \ln\left(\frac{a_1}{a_{now}}\right) &= \ln\left(\frac{\rho_{now}}{\rho_D}\right)^{\frac{1}{3}}\left(\frac{\rho_D}{\rho_C}\right)^{\frac{1}{4}}\left(\frac{\rho_C}{\rho_B}\right)^{\frac{1}{3}} \\ &= \ln\left(\frac{T_{now}}{T_D}\right)^{\frac{4}{3}}\left(\frac{T_D}{T_C}\right)\left(\frac{T_C}{T_B}\right)^{\frac{4}{3}}. \end{aligned} \quad (15)$$

After the tachyon condenses the single tachyon field rolls towards infinity with almost zero effective mass. When $\langle T \rangle$ becomes large, the tachyon field can be replaced by an equivalent scalar field $\sigma = \frac{4\sqrt{V_0}}{M_s} e^{-\frac{T}{2\sqrt{2}M_s}}$, which approaches zero when $T \rightarrow \infty$. With the new field the action can be expanded around $\sigma \sim 0$ and to first order in λ ,

$$L_{\phi-\sigma} = -\frac{1}{2}\sigma^2 - \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \lambda\phi^2\sigma^2, \quad (16)$$

i.e. at point C' in Fig.1, with

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{1}{a^2} + \frac{8\pi}{3M_p^2} \left[\frac{1}{2}\sigma^2 + \frac{1}{2}m^2\phi^2 - \lambda\sigma^2\phi^2 + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\sigma}^2 \right]. \quad (17)$$

Scalar fields, σ and ϕ , then take on similar equations of motion:

$$\ddot{\sigma} + 3H\dot{\sigma} + \left[\left(\frac{M_s}{2}\right)^2 - 2\lambda\phi^2 \right] \sigma = 0 \quad (18)$$

$$\ddot{\phi} + 3H\dot{\phi} + (m^2 - 2\lambda\sigma^2) \phi = 0. \quad (19)$$

From 17 we can deduce that at C' , $\langle \sigma^2 \rangle$ and $\langle \phi^2 \rangle$ are at their minima, and dividethe energy density equally:

$$\langle \phi^2 \rangle_{min} = \frac{3M_p^2}{8\pi m^2 a^2}. \quad (20)$$

During the subsequent contraction phase, as scale factor a diminishes, $\langle \sigma^2 \rangle$ and $\langle \phi^2 \rangle$ blue-shift, leading to a decrease of the effective masses. Considering that $m_\sigma^2 = (\frac{m_s}{2})^2 - 2\lambda\phi^2$, m_σ^2 decreases to zero and σ loses its validity and one should returns to T . At B' in Fig. 1, the contraction phase turns into deflation. We denote this point as the ending of contraction phase with the amplitude of ϕ given by:

$$\langle \phi^2 \rangle_e = \frac{m_s^2}{8\lambda}. \quad (21)$$

E-foldings can be computed accordingly, $N_c = \frac{1}{3} \ln \frac{\langle \phi^2 \rangle_e}{\langle \phi^2 \rangle_{min}}$, because $\langle \phi^2 \rangle \propto a^3$. Upon substituting the current size of universe $a_{current} \sim 5.4 \times 10^{61} l_p$ into N_c , we push back the current universe to the beginning of the deflation era:

$$N_c = \frac{1}{3} \ln \frac{8\pi m^2 m_s^2 a^2}{8\lambda 3M_p^2} \sim 118, \quad (22)$$

where $m \sim \frac{1}{10}m_s \sim \frac{1}{100}M_p$, $\lambda \sim 0.25$.

It is convincing then to say that, with a contraction which can generate 120 e-foldings, the expansion era after the bounce can enjoy an analogous number of e-foldings[15]. We can conclude that our bounce model solves the flatness problem by the 118 e-foldings in the contraction phase.

Conclusion and Outlook

We present a viable alternative to inflation based on string theory utilising coupled scalar and tachyon fields in the D- \bar{D} -brane system [6]. The CSTB cosmos undergoes an extended period of tachyon matter dominated contraction in which a stable and scale invariant spectrum of primordial density perturbations is produced [7]. The model has a signature out-of-thermal-equilibrium production of dark matter which can be tested by the future array of dark matter detections [8], independently of cosmological observations. Exploration on matter production and baryon asymmetry genesis is well underway [29, 30]. We expect our success can inspire further studies on bounce models built from string theory or other quantum gravity theories.

Recently an interesting attempt is made to use AdS/CFT correspondence to study the evolution of matter through the bounce point [32], extending the study AdS singularity [33, 34, 35, 36, 37, 38, 39, 40] to the Bounce Universe. Together with earlier initiatives of using AdS/CFT to study cosmic singularities [41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52] this holds great promise of a thorough study of bounce universe physics.

Another interesting question to ask if the CST-Bounce model can be embedded in higher dimensions, like [31], and to study the stabilisation of the extra moduli in this context. Further explorations shall also exploit the symmetries and dualities of the mother string theory as in [9, 11] as string-cosmology models utilising T-duality hold more promise of modulus stabilisation [53, 54, 55].

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